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THE HOLOMORPHIC FUNCTIONAL CALCULUS

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INTRODUCTION.

The classic holomorphic functional calculus was invented thirty years ago by Arens and Calderón [2]. Since then, it has proven to be an invaluable tool in the study of Banach algebras. It has also attracted a great deal of attention in itself, and many versions and alternative proofs have appeared.

Craw [3] produced the first version of a global functional calculus. By this we mean a morphism applying every function holomorphic near the spectrum of an algebra onto an element of the algebra.

In this paper we give another presentation of the global functional calculus. Our proof differs from Craw's, and also from Taylor's [5], in that we make no use of the classic functional calculus.

We start off by considering finitely determined open sets in §1, and holomorphic functions defined on the topological dual of an algebra in §2. In §3 a notion similar to polynomial convexity is introduced. We then describe, in §4, the set of germs of holomorphic functions over the spectrum as a direct limit of sets of holomorphic functions over open polynomially convex subsets of $\mathbf{C}^n$, and give our version of the functional calculus in §5. Throughout, A denotes a complex commutative unitary Banach algebra.

§1. We shall consider the topological dual A' of A with the weak *-topology. Thus, if γ_0 is an element of A' there is a basis for neighborhoods of γ_0 made up of sets like

$$U_{\gamma_0} = \{\gamma \in A' : |\gamma(a_i)-\gamma_0(a_i)| < 1, \ i = 1,\dots,n\} .$$

The elements $a_1,\dots,a_n$ may be chosen to be linearly independent. Once this is done, define

$$u = \hat{a}_1 \times \dots \times \hat{a}_n : A' \longrightarrow \mathbf{C}^n \qquad (u(\gamma) = (\gamma(a_1),\dots,\gamma(a_n)))$$

u is a linear continuous function, and because of the linear independence of the a_i, it is onto, and therefore open. Hence $u(U_{\gamma_0})$ is the open polydisc of $\mathbf{C}^n$ centered in $u(\gamma_0)$, and with radius one. Note that for γ to belong to U_{γ_0}, only its behaviour over $a_1,\dots,a_n$ is relevant. We say that U_{γ_0} is finitely determined by $a_1,\dots,a_n$, or by u.

Now if W is any open set in A', we say it is finitely determined by $v = \hat{b}_1 \times \dots \times \hat{b}_k$ (the b_i are independent) if $W = v^{-1}(v(W))$. Of course the non-trivial inclusion is $v^{-1}(v(W)) \subset W$, which says that if γ behaves over $b_1,\dots,b_k$ as an element of W, then γ belongs to W. For any open set U in $\mathbf{C}^k$, $v^{-1}(U)$ is finitely determined by v. We think of W as an infinite cilinder over the open set $v(W)$ of $\mathbf{C}^k$.

Different uples may determine the same open set; for example A' is determined by any uple. We need to partially order the uples (or the u's) determining a given W in A'. This will be done as follows: $u \leqslant v$ when the diagram

$$\begin{tikzcd} & v(W) \arrow[d, "\pi_{kn}"] \\ W \arrow[ur, "v"] \arrow[r, "u"'] & u(W) \end{tikzcd}$$

commutes. Here π_{kn} is the projection to the first n coordinates.

We shall need the following facts about finitely determined open sets.

PROPOSITION. *Let* W' *be finitely determined, and* W *finitely determined by* u. *Then there is a* $v \geqslant u$ *which determines both* W *and* W'.

Proof. If $u = \hat{a}_1 \times \dots \times \hat{a}_n$ and W' is finitely determined by $b_1,\dots,b_k$, let F be the subspace of A generated by $a_1,\dots,a_n,b_1,\dots,b_k$. Let $a_1,\dots,a_n,a_{n+1},\dots,a_m$ be a basis of F, and put $v = \hat{a}_1 \times \dots \times \hat{a}_m$.

The following facts are elementary.

PROPOSITION. *Finite unions of finitely determined open sets are finitely determined open sets.*

PROPOSITION. *All compact sets of* A' *have a basis for neighborhoods which are finitely determined open sets.*

§2. We now consider holomorphic functions over open subsets of A'. We say $f\colon U \longrightarrow \mathbf{C}$ is holomorphic when

i) the complex directional derivatives

$$\lim_{\lambda\to 0} \frac{f(x+\lambda y) - f(x)}{\lambda} \quad \text{exist for all } x \text{ in } U \text{ and } y \text{ in } A'.$$

ii) f is locally bounded.

The set O(U) of such functions over U form an algebra. Note that the Gelfand transforms of elements of A belong to O(A'). We call ß the subalgebra of O(A') which these elements generate.

The local boundedness condition which we ask of these functions makes them depend locally on just finite variables: every point in A' has a finitely determined neighborhood, that is, an infinite cilinder with a finite dimensional base. As we move in this cilinder, only a finite number of variables are bounded. The functions, holomorphic and bounded, must be constant as of the rest of the variables. To state this more clearly, we have the following

PROPOSITION. *The following are equivalent:*

i) $f\colon U \longrightarrow \mathbf{C}$ *is holomorphic.*

ii) *For every* $\gamma \in U$ *there are: a neighborhood* U_γ *of* γ, *linearly independent elements* $a_1,\dots,a_n$ *of* A, *and* $F \in O(u(U_\gamma))$ *such that* $f = Fu$ *over* U_γ; *where* $u = \hat{a}_1 \times \dots \times \hat{a}_n$.

Proof. (Allan, [1])

i) ⇒ ii) Let $\gamma \in U$. There is a neighborhood

$$U_{\gamma_0} = \{\gamma \in A' : |\gamma(a_i)-\gamma_0(a_i)| < 1, \quad i = 1,\dots,n\} \subset U$$

with $a_1,\dots,a_n$ linearly independent, over which f is bounded. Set $u = \hat{a}_1 \times \dots \times \hat{a}_n : A' \longrightarrow \mathbf{C}^n$. u is continuous, linear and open. We want to define a function $F \in O(u(U_{\gamma_0}))$ such that $Fu = f$. Let

$$F(z) = f(\gamma) \quad , \quad \text{if } z = u(\gamma)$$

F is well defined: suppose $\gamma, \gamma' \in U_{\gamma_0}$ with $u(\gamma) = u(\gamma')$. For all $\lambda \in \mathbf{C}$, $u(\gamma'-\gamma_0) = u(\lambda\gamma+(1-\lambda)\gamma'-\gamma_0)$, so

$$|\lambda\gamma(a_i)+(1-\lambda)\gamma'(a_i)-\gamma_0(a_i)| = |\gamma'(a_i)-\gamma_0(a_i)| < 1, \; i = 1,\dots,n.$$

Therefore $\lambda\gamma+(1-\lambda)\gamma' \in U_{\gamma_0}$ for every $\lambda \in \mathbf{C}$, then

$$\alpha\colon \mathbf{C} \longrightarrow \mathbf{C} \;, \quad \alpha(\lambda) = f(\lambda\gamma+(1-\lambda)\gamma')$$

is an entire bounded function; so it is constant.

$$f(\gamma') = \alpha(0) = \alpha(1) = f(\gamma).$$

The continuity and the existence of partial derivatives of F is easily verified, so by Hartog's theorem, F is holomorphic.

ii) $\Rightarrow$ i) is simple.

In the situation of the proposition, we shall say that f is finitely determined by $a_1,\dots,a_n$, or by u, over U_{γ_0}.

If W is a finitely determined open subset of A', and $f \in O(W)$, we say f is finitely determined by u over W if W is finitely determined by u and there is an $F \in O(u(W))$ such that $f = Fu$ over W. Finitely determined functions of O(W) form a subalgebra which we denote F(W). It is easily verified that the following holds.

PROPOSITION. *If* W *is finitely determined and* $f \in O(W)$ *is bounded, then any* u *that determines* W, *determines* f.

There are, however, unbounded elements in F(W). To clarify the structure of F(W), consider for u and v determining W with $u \leqslant v$,

$$f_{uv}: O(u(W)) \longrightarrow O(v(W)), \quad f_{uv}(g) = g\pi_{kn} .$$

These f_{uv} are a direct system and it is not hard to verify that $F(W) = \varinjlim_{u} O(u(W))$: the mappings $O(u(W)) \longrightarrow F(W)$ given by $f \longrightarrow fu$ induce a map $\varinjlim_{u} O(u(W)) \longrightarrow F(W)$ which is an isomorphism.

We shall consider O(u(W)) endowed with the topology of uniform convergence over compact subsets of u(W), and F(W) with the direct limit topology. This topology is finer than the topology of uniform convergence on compact subsets of W.

§3. We need in A', a notion analogous to the notion of polynomial convexity in $\mathbf{C}^n$. Define, for each subset B of A',

$$\widetilde{B} = \{\gamma \in A' : |P(\gamma)| \leqslant \sup_{b\in B} |P(b)| , \quad \text{for all } P \in \beta\}$$

We say B is strongly β-convex if $\widetilde{B} = B$, and β-convex if $\widetilde{K} \subset B$ for all compact subsets K of B. Note that the spectrum of A, X(A), is strongly β-convex: if $\gamma \in \widetilde{X(A)}$, let $a,b \in A$ and $P_{ab} = \hat{a}\hat{b}-\widehat{ab} \in \beta$.

$$|P_{ab}(\gamma)| \leqslant \sup_{X(A)} |P_{ab}| = 0$$

so $\gamma(a)\gamma(b)-\gamma(ab) = 0$ for all $a,b \in A$, and $\gamma \in X(A)$.

For finitely determined open subsets of A', β-convexity and polynomial convexity are related as follows.

PROPOSITION. *Let* W *be open in* A', *finitely determined by* $u = \hat{a}_1 \times \dots \times \hat{a}_n$. *Then the following are equivalent:*

i) W *is* β-*convex.*

ii) u(W) *is polynomially convex.*

Proof. i) $\Rightarrow$ ii) Let H be compact, contained in u(W). We must show that its polynomially convex hull $\hat{H}$ is a subset of u(W).

Define $\sigma: \mathbf{C}^n \longrightarrow A'$ by $\sigma(z) = \sum_{i=1}^{n} z_i\phi_i$, where $\phi_i(a_j) = \delta_{ij}$ and $\phi_i = 0$ over the rest of a basis B for A that extends $a_1,\dots,a_n$. Then $u\sigma$ is the identity over $\mathbf{C}^n$, and σ is continuous. Let $K = \sigma(H)$. K is compact, $u(K) = H$ and $K \subset u^{-1}(u(W)) = W$.

We must verify, then, that $\widehat{u(K)} \subset u(W)$. Since $\tilde{K} \subset W$, $u(\tilde{K}) \subset u(W)$ and it will be enough to show that $\widehat{u(K)} \subset u(\tilde{K})$. Let $z_0 \in \widehat{u(K)}$. Then

$$|P(z_0)| \leqslant \sup_{u(K)} |P(z)| \text{ for all } P \in \mathbf{C}[X_1,\dots,X_n]$$

Now let $\gamma_0 = \sigma(z_0)$. $u(\gamma_0)=z_0$, and we must see that $\gamma_0 \in \tilde{K}$, that is,

$$|Q(\gamma_0)| \leqslant \sup_{\gamma\varepsilon K} |Q(\gamma)| \text{ for all } Q \in \beta$$

It is not true that, given $Q \in \beta$, there is a $P \in \mathbf{C}[X_1,\dots,X_n]$ with $Q = Pu$. However, there is a polynomial $P \in \mathbf{C}[X_1,\dots,X_n]$ which makes the equality valid over $\sigma(\mathbf{C}^n)$, which is what we really need.

To show the existence of such P, say $\hat{b}_1,\dots,\hat{b}_k$ are the "coordinates" appearing in Q. Then there are $a_1,\dots,a_m$ in B, which generate all b_j and amongst which we may find $a_1,\dots,a_n$. There is a polynomial $\overline{P} \in \mathbf{C}[X_1,\dots,X_m]$ for which $Q = \overline{P}(\hat{a}_1,\dots,\hat{a}_m)$. Then

$$Q(\sigma(z)) = \overline{P}(\hat{a}_1,\dots,\hat{a}_m)(\sigma(z)) = \overline{P}(\sigma(z)(a_1),\dots,\sigma(z)(a_m))=$$

$$= \overline{P}(\sigma(z)(a_1),\dots,\sigma(z)(a_n),0,\dots,0)$$

Let $P(X_1,\dots,X_n) = \overline{P}(X_1,\dots,X_n,0,\dots,0)$. Then $Q = Pu$ over $\sigma(\mathbf{C}^n)$, and

$$|Q(\gamma_0)| = |P(u(\gamma_0))| = |P(z_0)| \leqslant \sup_{\gamma\varepsilon K} |P(u(\gamma))| = \sup_{\gamma\varepsilon K} |Q(\gamma)|.$$

Therefore $\gamma_0 \in \tilde{K}$.

ii) $\Rightarrow$ i) If K is a compact subset of W, let $H = u(K)$. H is compact, contained in u(W), so $\hat{H} \subset u(W)$. We want to show that $\tilde{K} \subset W$. Since $u^{-1}(\hat{H}) \subset u^{-1}(u(W)) = W$, it will be enough to see $\tilde{K} \subset u^{-1}(\hat{H})$, that is, $u(\tilde{K}) \subset \hat{H}$. This is easily verified once we note that $Pu \in \beta$ for all $P \in \mathbf{C}[X_1,\ldots,X_n]$.

In the preceding proof we have shown the validity of the equality $\widehat{u(K)} = u(\widetilde{K})$, for compact sets $K = \sigma(H)$, with H compact in $\mathbf{C}^n$. This equality for arbitrary compact subsets of A' is false. For example, we know the spectrum $X(A)$ is a compact β-convex subset of A', but $u(X(A)) = sp(a_1,\ldots,a_n)$ is not, in general, polynomially convex.

This fact is an important setback in the construction of a holomorphic functional calculus, for $sp(a_1,\ldots,a_n)$ will not have a basis for neighborhoods whose elements are polynomially convex open subsets of $\mathbf{C}^n$. In the classical functional calculus, this difficulty is overcome by the Arens-Calderón trick. In this version, what we need is the following.

PROPOSITION. *Let* K *be a compact* β*-convex subset of* A'. *Then* K *has a basis for neighborhoods made up of* β*-convex, finitely determined open sets.*

Proof. K has a basis for neighborhoods made up of finitely determined open sets. Let W be such a neighborhood, determined by $u = \hat{a}_1 \times \ldots \times \hat{a}_n$. Also, let $c > 0$ be such that $K \subset D = \{\gamma \in A' : \|\gamma\| \leq c\}$.

Given $P \in \beta$, let $K_P = \{\gamma \in A' : |P(\gamma)| \leq \sup_K |P|\}$. K is β-convex, so $K = \bigcap_P K_P$. Since $D \cap K_P$ is compact for each P, there are $P_1,\ldots,P_k$ with

$$K \subset D \cap K_{P_1} \cap \ldots \cap K_{P_k} \subset W.$$

Let $v = \hat{a}_1 \times \ldots \times \hat{a}_n \times \ldots \times \hat{a}_m \geq u$, such that v determines P_i for $i = 1,\ldots,k$; that is, there are polynomials $Q_1,\ldots,Q_k \in \mathbf{C}[X_1,\ldots,X_m]$ with $P_i = Q_i v$. Let

$$v(K)_{Q_i} = \{z \in \mathbf{C}^m : |Q_i(z)| \leq \sup_{v(K)} |Q_i|\}$$

For every i, this set is polynomially convex and $v(K_{P_i}) \subset v(K)_{Q_i}$. Put

$$K_0 = v(D) \cap v(K)_{Q_1} \cap \ldots \cap v(K)_{Q_k}.$$

K_0 is a polynomially convex compact set, for D is compact and $v(D)$ is polynomially convex: to see this let V be the subspace of A generated by $a_1,\ldots,a_m$. Its dual V' may canonically be thought of as a quotient of A'. Factoring v through this quotient we obtain an isomorphism $\overline{v}: V' \longrightarrow \mathbf{C}^m$. We may then identify $D' = \{x \in V' : \|x\| \leq c\}$ with $\overline{v}(D') = v(D)$. Now if $z \in \mathbf{C}^m - v(D)$, $z = \overline{v}(x)$ with $\|x\| > c$. Let

L: $V' \longrightarrow \mathbf{C}$ be linear, with norm one and such that $|L(x)| = \|x\|$, and $Q = L\bar{v}^{-1}: \mathbf{C}^m \longrightarrow \mathbf{C}$. $Q \in \mathbf{C}[X_1,\ldots,X_m]$ and

$$|Q(z)| = |L\bar{v}^{-1}\bar{v}(x)| = |L(x)| = \|x\| > c = \sup_{D'}|L| = \sup_{v(D)} |Q|$$

Therefore for all $z \in \mathbf{C}^m - v(D)$ there is a Q with $|Q(z)| > \sup_{v(D)} |Q|$; $v(D)$ is polynomially convex.

We also have $v(K) \subset K_0 \subset v(W)$. The first inclusion because $K \subset D \cap K_{P_1} \cap \ldots \cap K_{P_k}$ implies

$$v(K) \subset v(D \cap K_{P_1} \cap \ldots \cap K_{P_k}) \subset v(D) \cap v(K)_{Q_1} \cap \ldots \cap v(K)_{Q_k} = K_0$$

and to verify the second, let $z \in K_0$, $\gamma \in D$ with $z = v(\gamma)$. We have

$$|P_i(\gamma)| = |Q_i v(\gamma)| = |Q_i(z)| \leqslant \sup_{v(K)} |Q_i| = \sup_{K} |P_i|$$

for $i = 1,\ldots,k$; that is, $\gamma \in D \cap K_{P_1} \cap \ldots \cap K_{P_k} \subset W$, and $z \in v(W)$.

Now let U be a polynomially convex open subset of $\mathbf{C}^m$ such that $v(K) \subset K_0 \subset U \subset v(W)$. Then $K \subset v^{-1}(U) \subset v^{-1}(v(W)) = W$, and $v^{-1}(U)$ is a finitely determined open subset of A', and it is β-convex thanks to the preceding proposition.

§4. We return now to holomorphic functions over A'. If we have two open sets $U \subset V$, we also have the restriction mapping $O(V) \longrightarrow O(U)$. Fix a compact subset K of A'. Its open neighborhoods are partially ordered and the restriction mappings form a direct system. Therefore $O(K) = \varinjlim O(U)$ is defined.

However, as we have seen before, holomorphic functions are locally finitely determined, so the same happens to holomorphic functions over a sufficiently small neighborhood of a compact set. We have, in fact:

PROPOSITION. *Let* K *be a compact* β*-convex subset of* A'. *Then* $O(K) = \varinjlim F(W)$, *where* W *are open, finitely determined,* β*-convex neighborhoods of* K.

Proof. Say $W' \subset W$, $f \in F(W)$, and u determines f and W. Then there is a $v \geqslant u$ determining both W and W'.

$$\begin{tikzcd} & v(W) \arrow[d, "\pi_{mn}"] & \\ W \arrow[ur, "v"] \arrow[r, "u"'] & u(W) \arrow[r, "F"'] & \mathbf{C} \end{tikzcd}$$

If $f = Fu$, let $F' = F\pi_{mn}$. Then $F'v = f$ over W, and therefore over W'. This defines maps

$$F(W) \longrightarrow F(W')$$

which form a direct system, so $\varinjlim_{W} F(W)$ is defined. The maps

$$F(W) \longrightarrow O(W) \longrightarrow O(K)$$

induce a morphism

$$\varinjlim_{W} F(W) \longrightarrow O(K)$$

which is easily seen to be an isomorphism.

So if K is a compact β-convex subset of A', $O(K)$ may be thought of as a direct limit of algebras $O(u(W))$, where $u(W)$ are open polynomially convex subsets of $\mathbf{C}^n$. We consider $O(K)$ with the direct limit topology.

§5. We are now ready for our main theorem.

THEOREM. *Let* A *be a commutative Banach algebra. There is a unique continuous unitary algebra homomorphism*

$$E\colon O(X(A)) \longrightarrow A$$

with $E(\hat{a}) = a$.

Proof. The spectrum $X(A)$ is compact and β-convex, so we have $O(X(A)) = \varinjlim_{W}(\varinjlim_{u} O(u(W)))$, where $u(W)$ are open polynomially convex neighborhoods of $u(X(A)) = sp(a_1,\ldots,a_n)$, if $u = \hat{a}_1 \times \ldots \times \hat{a}_n$. Therefore all holomorphic functions over $u(W)$ are uniformly approximable by polynomials on the compact subsets of $u(W)$ [4]. Also, $O(u(W))$ induces a topology on $\mathbf{C}[X_1,\ldots,X_n]$ for which the unitary algebra homomorphism defined by $X_i \longmapsto a_i$ is continuous. We then have continuous unitary algebra homomorphisms

$$O(u(W)) \longrightarrow A$$

It is a purely technical matter to verify that these maps induce a map

$$O(X(A)) \longrightarrow A$$

with the required properties.

It is also easy to see that for every $f \in O(X(A))$, f and $\widehat{E(f)}$ coincide over $X(A)$, though not, in general, as elements of $O(X(A))$. We have found the following proposition useful.

PROPOSITION. *Let* $N = \{f \in O(X(A)) : f|X(A) = 0\}$. *Then* $E(N) = \mathrm{Rad}(A)$ *and* $N = E^{-1}(\mathrm{Rad}(A))$.

Proof. If $f \in N$, $\widehat{E(f)}|X(A) = f|X(A) = 0$, so $E(f) \in \mathrm{Rad}(A)$. If $a \in \mathrm{Rad}(A)$, $\hat{a} \in N$, and $E(\hat{a}) = a$. For the other equality, we already have $N \subset E^{-1}(\mathrm{Rad}(A))$, and if $E(f)$ belongs to the radical, $f|X(A) = \widehat{E(f)}|X(A) = 0$, so $f \in N$.

Note that when A is semisimple, the proposition says that N is the kernel of E.

The homomorphism defined by the theorem is, of course, the same as Craw's [3], the only possible difference being in the topologies of $O(X(A))$. In [3] $O(X(A))$ is presented as $\varinjlim_{U} H^\infty(U)$, where U are open neighborhoods of the spectrum and $H^\infty(U)$ the set of holomorphic functions over U which are bounded, with the supremum norm. Actually, the topologies are the same:

PROPOSITION. $\varinjlim_{U} H^\infty(U) \cong \varinjlim_{W,u} O(u(W))$.

Proof. As we have shown before, the open neighborhoods U of X(A) may be taken to be finitely determined and β-convex, for these form a basis for neighborhoods of X(A).

All maps $H^\infty(W) \longrightarrow \varinjlim O(u(W))$ are continuous, for if $f_n \longrightarrow 0$ in $H^\infty(W)$, all may be written as $F_n u$ (the same u, since the f_n are all bounded) and $F_n \longrightarrow 0$ uniformly over all of $u(W)$, not just compact subsets. Therefore

$$\varinjlim H^\infty(U) \longrightarrow \varinjlim O(u(W))$$

is continuous.

Now fix u, W, and suppose $F_n \longrightarrow 0$ in $O(u(W))$. Let Q be a compact neighborhood of $u(X(A))$, contained in $u(W)$. Then $F_n \longrightarrow 0$ uniformly over Q, hence over Q^0. Let $U = u^{-1}(Q^0)$. U is a neighborhood of X(A), and $f_n = F_n u \longrightarrow 0$ uniformly over U. So

$$\varinjlim O(u(W)) \longrightarrow \varinjlim H^\infty(U)$$

is also continuous.

The authors have found the presentation $O(K) = \varinjlim O(u(W))$ more manageable, for example in the following setting. Suppose F is a complex homogeneous space, not contained in $\mathbf{C}^n$. We want to define the set of germs of holomorphic functions defined near X(A), with images in F. This can be done considering for each finitely determined β-convex open neighborhood, W, of X(A) and each u that determines it, the set $O(u(W),F)$ with the compact-open topology, and then taking

$$O(X(A),F) = \lim_{\rightarrow} O(u(W),F).$$

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