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ON HAMILTON CIRCUITS IN CAYLEY DIGRAPHS OVER GENERALIZED DIHEDRAL GROUPS

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ABSTRACT. In this paper we prove that given a generalized dihedral group D_H and a generating subset S, if $S \cap H \neq \emptyset$ then the Cayley digraph $\vec{Cay}(D_H, S)$ is Hamiltonian. The proof we provide is via a recursive algorithm that produces a Hamilton circuit in the digraph.

1. Introduction

The *Cayley digraph* on a group G with generating set S, denoted $\overrightarrow{Cay}(G; S)$, is the digraph with vertex set G, and arc set containing an arc from g to gs whenever $g \in G$ and $s \in S$ (if we ask $S = S^{-1}$ and $e \notin S$, we have just a Cayley graph). Cayley (di)graphs of groups have been extensively studied and some interesting results have been obtained (see [3]). In particular, several authors have studied the following folk conjecture: every Cayley graph is Hamiltonian (see [4]). Another interesting problem is to characterize which Cayley digraphs have Hamiltonian paths. These problems tie together two seemingly unrelated concepts: traversability and symmetry on (di)graphs.

Both problems had been attacked for more than fifty years (started with [5]), yet not much progress has been made and they remain open. Most of the results proved thus far depend on various restrictions made either on the class of groups dealt with or on the generating sets (for example one can easily see that Cayley graphs on Abelian groups have Hamilton cycles). The class of groups with cyclic commutator subgroups has attracted attention of many researchers (see [2]). And for many technical reasons the key to proving that every connected Cayley graph on a finite group with cyclic commutator subgroup has a Hamilton cycle very likely lies with dihedral groups.

Given a finite abelian group H, the generalized dihedral group over H is

$$D_H = \left\langle H, \tau : \tau^2 = e \qquad \tau h \tau = h^{-1} \quad \forall h \in H \right\rangle$$

Recently (2010) in [1], working on generalized dihedral groups, it was proved that every Cayley graph on the dihedral group D_{2n} with n even has a Hamilton cycle. In 1982 in [7] Dave Witte proved that if $\vec{Cay}(D_H, S \cap H\tau)$ is Hamiltonian, then

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$\overrightarrow{Cay}(D_H, S)$ is also Hamiltonian. In this paper we take this result a step further and prove via a recursive algorithm that if $S \cap H \neq \emptyset$, then $\overrightarrow{Cay}(D_H, S)$ is Hamiltonian independently of what happens with $\overrightarrow{Cay}(D_H, S \cap H\tau)$.

In [6] S. Curran proved that if D_n is a dihedral group and $S = \{\tau, \rho_1\tau, \ldots, \rho_m\tau\}$ is a generating set which only contains reflections then if $\overrightarrow{Cay}(\mathbb{Z}_n, S')$ has a Hamilton circuit when $S' = \{\rho_1, \ldots, \rho_m\}$ then $\overrightarrow{Cay}(D_n, S)$ has a Hamilton circuit. We will show in this paper that his proof can be easily extended to generalized dihedral groups.

2. Preliminaries

We start by giving some known definitions.

Definition 1. *Given a finite abelian group H, the generalized dihedral group is defined as follows:*

$$D_H = \left\langle H, \tau : \tau^2 = e \qquad \tau h \tau = h^{-1} \quad \forall h \in H \right\rangle$$

Definition 2. *Given a group G and a generating subset $S \subseteq G$ such that $e \notin S$, we define the Cayley digraph $\overrightarrow{Cay}(D_H, S)$ as the digraph with vertex set G and arc from g to gs for every $g \in G$ and $s \in S$.*

In this paper we will describe a walk by giving its starting vertex and the list of generators that gives us the arcs of the walk in the order that the arcs are used, i.e. $(a; g_1 g_2 \ldots g_k)$ will denote the walk that goes through the vertices: a, ag_1, ag_1g_2, $\ldots$, $ag_1g_2\ldots g_k$. When we talk about arcs in a walk we only consider them in the direction used in the walk, even if they are edges. This means that if there is an edge between a and ag_i (this is an undirected link), and $a \to ag_i$ is in the walk, then in the walk we consider the edge as if it were an arc from a to ag_i. If $C = (a; g_1 \ldots g_k)$ then the length of C is k.

Definition 3. *We will say that a walk* ***covers*** *a set of vertex B if for every $b \in B$ there is an i such that $b = ag_1 \ldots g_i$.*

Definition 4. *A* ***circuit*** *is a walk with no repeated vertices other than the starting and the ending vertices.*

It is known that in a Cayley digraph given the list of generators we can start the circuit in any vertex and that if $g_1 \ldots g_k$ gives a circuit, so does $g_j g_{j+1} \ldots g_n g_1 g_2 \ldots g_{j-1}$. We will use this fact in the proof of the main result.

We now introduce some new properties that will be useful to prove our main result.

Definition 5. *A circuit in $\overrightarrow{Cay}(D_H, S)$ satisfies the h-property if it has an arc $a \to b$ such that $a, b \in H$.*

Definition 6. *A circuit in* $\overrightarrow{Cay}(D_H, S)$ *satisfies the* $h\tau$*-property if it has an arc* $a\tau \to b\tau$ *such that* $a, b \in H$*.*

Proposition 7. *Let* $g_1, g_2, \dots, g_k$ *be a list of generators that produces a circuit and let* $h_1, h_2 \in H$*. Then*

(1) *The circuit* $(h_1; g_1 g_2 \dots g_k)$ *satisfies the* h*-property if and only if the circuit* $(h_2\tau; g_1 g_2 \dots g_k)$ *satisfies the* $h\tau$*-property.*
(2) *The circuit* $(h_1; g_1 g_2 \dots g_k)$ *satisfies the* $h\tau$*-property if and only if the circuit* $(h_2\tau; g_1 g_2 \dots g_k)$ *satisfies the* h*-property.*

Proof. Suppose that the circuit $(h_1; g_1 g_2 \dots g_k)$ satisfies the h-property, this means that there is an arc $a \to b$ with $a, b \in H$. This arc is given by a generator g_i, this means that $ag_i = b$ but $a, b \in H$ and thus $g_i \in H$. Even more, $a = h_1 g_1 \dots g_{i-1}$, and, as $h_1 \in H$, so does $g_1 \dots g_{i-1}$. Take now $a' = h_2 \tau g_1 \dots g_{i-1}$, we know that $h_2\tau \in H\tau$, $g_1 \dots g_{i-1} \in H$ and $g_i \in H$, because of this $a' \in H\tau$ and $a' g_i \in H\tau$. And thus the circuit $(h_2\tau; g_1 g_2 \dots g_k)$ satisfies the $h\tau$-property. The proofs of the other implications are analogous. ☐

Corollary 8. *Let* $g_1, g_2, \dots, g_k$ *be a list of generators that produces a circuit and let* $h_1, h_2 \in H$*. Then*

(1) If the circuit $(h_1; g_1 g_2 \dots g_k)$ satisfies the h-property or the $h\tau$-property then the circuit $(h_2; g_1 g_2 \dots g_k)$ satisfies the h-property or the $h\tau$-property.
(2) If the circuit $(h_1\tau; g_1 g_2 \dots g_k)$ satisfies the h-property or the $h\tau$-property then the circuit $(h_2\tau; g_1 g_2 \dots g_k)$ satisfies the h-property or the $h\tau$-property.

Proof. This is easily proved applying Proposition 7 twice. ☐

Definition 9. *A circuit in* $\overrightarrow{Cay}(D_H, S)$ *satisfies the* τ*-property if it has an arc* $c \to c\tau$ *such that* $c \in H$*.*

Proposition 10. *Let* $g_1, g_2, \dots, g_k$ *be a list of generators that produces a circuit and let* $h_1, h_2 \in H$*. Then if* $(h_1; g_1 \dots g_k)$ *satisfies the* τ*-property then* $(h_2; g_1 \dots g_k)$ *satisfies the* τ*-property.*

Proof. Suppose that the circuit $(h_1; g_1 \dots g_k)$ satisfies the τ-property, this means that there is an arc $c \to c\tau$ with $c \in H$ in the circuit. Let g_i be the generator in the circuit that produces this arc, then $g_i = \tau$, and $h_1 g_1 \dots g_{i-1} \in H$, as $h_1 \in H$ so does $g_1 \dots g_{i-1}$. Take now the circuit $(h_2; g_1 \dots g_k)$ and let $c' = h_2 g_1 \dots g_{i-1}$, as $h_2 \in H$ so does c', and thus $c' \to c'\tau = c' g_i$ is an arc of the circuit, and the circuit satisfies the τ-property. ☐

Definition 11. *A circuit in* $(a; g_1 g_2 \dots g_k)$ *satisfies the* T*-property if* $g_k = \tau$*.*

Definition 12. *An* A*-circuit is a circuit that satisfies the* h*-property, the* $h\tau$*-property, the* τ*-property and the* T*-property.*

Lemma 13. *If* $\langle S \rangle = D_H$*, then* $S \cap H\tau \neq \emptyset$*. And without loss of generality we can suppose that* $\tau \in S$*.*

Proof. If $S \subseteq H$ then $\langle S \rangle \leq H < D_H$, then $S \nsubseteq H$, thus there is an element $s \in S$ such that $s \notin H$, but this means that $s \in H\tau$ and thus $S \cap H\tau \neq \emptyset$. Given $h\tau \in H\tau \cap S$ we will define an automorphism that maps $h\tau$ into τ:

$$\begin{aligned} \phi : D_H &\to D_H \\ \phi|_H &= I \\ \phi(a\tau) &= h^{-1}a\tau \end{aligned}$$

$\phi(H) = H$ and $\phi(H\tau) = H\tau$; this means that ϕ is a bijection. Let us see that $\phi(ab) = \phi(a)\phi(b)$.

If $a, b \in H$ this is true.

If $a \in H$ and $b = h_1\tau$

$$\phi(ah_1\tau) = h^{-1}ah_1\tau = ah^{-1}h_1\tau = \phi(a)\phi(h_1\tau).$$

If $a = h_1\tau$ and $b \in H$

$$\phi(h_1\tau b) = h^{-1}h_1\tau b = \phi(h_1\tau)\phi(b).$$

Finally, if $a = h_1\tau$ and $b = h_2\tau$, $ab = h_1h_2^{-1}$,

$$\begin{aligned} \phi(h_1h_2^{-1}) &= h_1h_2^{-1} \\ &= h_1\tau h_2\tau \\ &= h_1\tau hh^{-1}h_2\tau \\ &= h_1h^{-1}\tau h^{-1}h_2\tau \\ &= h^{-1}h_1\tau h^{-1}h_2\tau \\ &= \phi(h_1\tau)\phi(h_2\tau). \end{aligned}$$

As $\phi|_H = I$ and $\phi(h\tau) = h^{-1}h\tau = \tau$. Using this isomorphism we can transform $h\tau$ in τ, thus we can suppose without loss of generality that $\tau \in S$. $\square$

Lemma 14. *Let $S = \{\tau, s_1, \ldots, s_n\} \subseteq D_H$, $S_0 = \{\tau\}$, $S_k = \{\tau, s_1, \ldots, s_k\}$, and $G_k = \langle S_k \rangle \leq D_H$. Given $s_j \in S$, let $h \in H$ be such that $s_j = h\tau^\alpha$ with $\alpha \in \{0, 1\}$. Let φ be the least positive power of h such that $h^\varphi \in G_{j-1}$. Then G_{j-1}, hG_{j-1}, h^2G_{j-1}, …, $h^{\varphi-1}G_{j-1}$ is a partition of G_j.*

Proof. Let $s_i \in S_{j-1}$; then $s_is_j = s_ih\tau^\alpha = h^{t_i}s_i\tau^\alpha$ with $t_i = \pm1$, but as $\tau \in S_{j-1}$, $s_i\tau^\alpha \in G_{j-1}$. Thus $s_is_j = h^{t_i}b_i$ with $b_i \in G_{j-1}$. As any $a \in G_j$ is a word of generators, every time that s_j appears in the word we can write it as $h\tau^\alpha$, and then move the h to the left. This means that we can write $a = h^tb$, with $b \in G_{j-1}$, but as $h^\varphi \in G_{j-1}$ we may assume that $0 \leq t < \varphi$.

We have proven that $G_j = \bigcup_{r=0}^{\varphi-1} h^rG_{j-1}$. Let us prove that this is actually a partition.

Suppose that $h^{r_1}G_{j-1} \cap h^{r_2}G_{j-1} \neq \emptyset$, then $h^{r_1}a = h^{r_2}b$ with $a, b \in G_{j-1}$ and $0 \leq r_1, r_2 < \varphi$. This means that $h^{r_1-r_2} = ba^{-1} \in G_{j-1}$ and then $r_1 - r_2 = 0$ (as $-\varphi < r_1 - r_2 < \varphi$) and $r_1 = r_2$. Thus we have $h^{r_1}G_{j-1} = h^{r_2}G_{j-1}$. This proves that G_{j-1}, hG_{j-1}, h^2G_{j-1}, …, $h^{\varphi-1}G_{j-1}$ is a partition of G_j. $\square$

3. MAIN RESULT

Theorem 15. *If $H \cap S \neq \emptyset$, then $\overrightarrow{Cay}(D_H, S)$ has a Hamilton circuit.*

Remark 16. *Together with the proof we give a graphic example of how the algorithm works on $\overrightarrow{Cay}(D_{\mathbb{Z}_{30}}, \{\tau, r^6, r^{10}, r^{15}\tau\})$.*

Proof. We will give a recursive algorithm that finds the circuit, obtaining in each step an A-circuit with starting vertex in H that covers some subgraph.

Let $h \in H \cap S$.

Base Step: $\tau \in S$, let $S_1 = \{\tau, h\} \subseteq S$ and $G_1 = \langle S_1 \rangle \leq D_H$. We will show that $C_1 = (e; 2 * ((ord(h) - 1) * h, \tau))$ is a circuit that covers G_1. Let $a \in G_1$, if $a \in H$, then $a = h^r$, with $0 \leq r \leq ord(h) - 1$, and so a is the rth vertex of the circuit. If $a \in H\tau$ and $a \in G_1$, then

$$\begin{aligned} a &= h^r \tau \\ &= h^{ord(h)-1} h h^r \tau \\ &= h^{ord(h)-1} h^{r+1} \tau \\ &= h^{ord(h)-1} \tau h^{-r-1} \\ &= h^{ord(h)-1} \tau h^{ord(h)-r-1} \end{aligned}$$

and thus it if

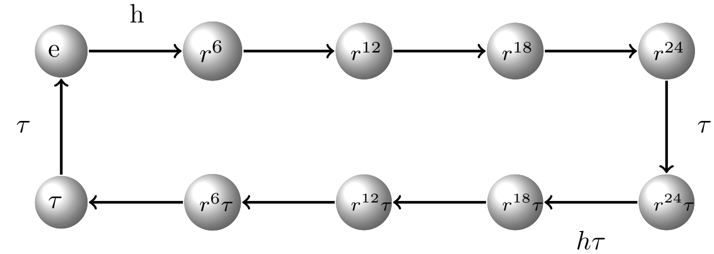
$$j = ord(h) - 1 + 1 + ord(h) - r - 1 = 2ord(h) - r - 1$$

then a is the jth vertex of the walk. We have seen that C_1 covers G_1. Also the last vertex of the walk is

$$e h^{ord(h)-1} \tau h^{ord(h)-1} \tau = \left(h^{ord(h)-1} \tau\right)^2 = e$$

and, as $|G_1| = 2ord(h)$ and the walk $(e; 2 * ((ord(h) - 1) * h, \tau))$ has the same length, C_1 is a circuit covering G_1. We will show now that it is an A-circuit. The last generator is τ and thus it satisfies the T-property. It has the arc $e \to h$ and the arc $(h)^{ord(h)-1} \tau \to (h)^{ord(h)-1} \tau h$, and so it satisfies the h-property and the $h\tau$-property. It has the arc $(h)^{ord(h)-1} \to (h)^{ord(h)-1} \tau$, and so it satisfies the τ-property. Then it is an A-circuit with starting vertex in H, because its starting vertex is e.

$G_1 = \langle \tau, r^6 \rangle$



Recursive Step: let $s_n \in S$, $S_n = S_{n-1} \cup \{s_n\} \subseteq S$, and $G_n = \langle S_n \rangle \leq D_H$. We divide this step in two cases, $s_n = h_n$ and $s_n = h_n\tau$. In each case we will find recursively a circuit covering each of the parts of G_n given by the second lemma and we will join all them, forming an A-circuit that covers G_n.

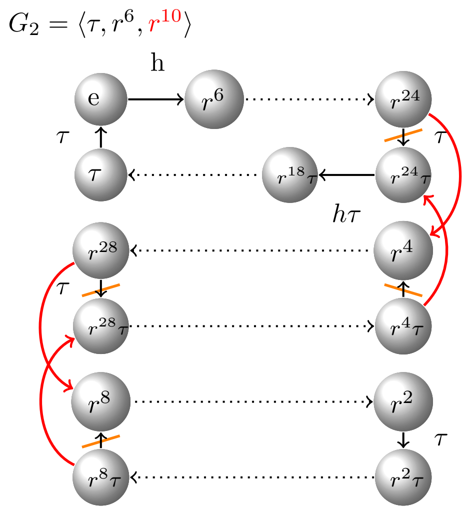
Case 1 ($s_n = h_n$): By the previous step we have $C_{n-1}(1) = C_{n-1}$ covering G_{n-1}, and this gives the base step of the second recursion. Suppose now that we found a circuit $C_{n-1}(k)$ covering $h_n^{k-1}G_{n-1}$ that satisfies the τ-property and the T-property with starting vertex belonging to H. As $C_{n-1}(k)$ satisfies the τ-property it has the arc $c_n \to c_n\tau$ with $c_n \in H$. We multiply c_n by h_n and then copy the circuit starting in c_nh_n, this gives us a circuit $C_{n-1}(k+1)$, and because this circuit has initial vertex in $h_n^kG_{n-1}$ and the generators of the circuit belong to G_{n-1}, this circuit covers $h_n^kG_{n-1}$. As c_nh_n and the starting vertex of $C_{n-1}(k)$ both belong to H and $C_{n-1}(k)$ satisfies the τ-property then because of Corollary 8 we have that $C_{n-1}(k+1)$ satisfies the τ-property. Because of the T-property of $C_{n-1}(k)$, the last arc of $C_{n-1}(k+1)$ is given by τ, and because $\tau^2 = e$ the last vertex from the circuit is $c_nh_n\tau$. Multiplying $c_nh_n\tau$ by h_n gives us $c_nh_n\tau h_n = c_nh_nh_n^{-1}\tau = c_n\tau$. We now have these arcs joining the circuits

$$c_n \to c_nh_n \quad \text{and} \quad c_nh_n\tau \to c_n\tau;$$

we use these arcs and erase the arcs:

$$c_n \to c_n\tau \quad \text{and} \quad c_nh_n\tau \to c_nh_n$$

to join the circuits. As the circuit satisfies the τ-property and its beginning vertex belong to H, we may repeat the process to find a circuit covering $h_n^{k+1}G_{n-1}$ and, as we did not use the arc giving the T-property of $C_{n-1}(k+1)$ we can join them. We repeat the process until we find a circuit covering the last part of G_n and join the circuits with the arcs given in the recursion. The arcs that gave us the h-property and the $h\tau$-property in $C_{n-1}(1)$ are still used in this circuit, we did not use the arc of the T-property from the last circuit, and the final arc is τ, so this is an A-circuit, and as we keep the starting vertex from $C_{n-1}(1)$, its starting vertex belongs to H.



Case 2 ($s_n = h_n\tau$): Again by the previous step we have $C_{n-1}(1) = C_{n-1}$ covering G_{n-1}, and this gives the base step of the second recursion. Suppose now that we found a circuit $C_{n-1}(k)$ covering $h_n^{k-1}G_{n-1}$ that satisfies the h-property and the $h\tau$-property. As $C_{n-1}(k)$ satisfies the h-property it has two vertices a_n and b_n from H that are neighbours. Multiply them both by s_n. As $a_n, b_n \in h^{k-1}G_{n-1}$ and

$$\begin{aligned} a_n s_n &= a_n h_n \tau \\ &= h_n a_n \tau \end{aligned}$$

and

$$\begin{aligned} b_n s_n &= b_n h_n \tau \\ &= h_n b_n s_n \end{aligned}$$

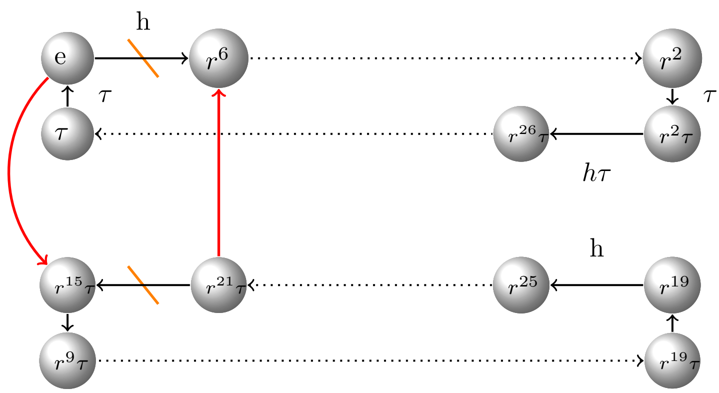
we have that both $a_n s_n$ and $b_n s_n$ belong to $h_n^k G_{n-1}$. As $a_n g_i = b_n$ with $g_i \in H$ we have that

$$\begin{aligned} b_n s_n g_i &= b_n h_n \tau g_i \\ &= h_n b_n g_i^{-1} \tau \\ &= h_n a_n \tau \\ &= a_n h_n \tau \\ &= a_n s_n, \end{aligned}$$

thus g_i gives an arc from $b_n s_n$ to $a_n s_n$ and if $C_{n-1}(k) = (c; g_1 \dots g_i \dots g_m)$ is the original circuit then $(b_n s_n; g_i \dots g_m g_1 \dots g_{i-1})$ is a circuit that covers $h_n^k G_{n-1}$. But then $C_{n-1}(k+1) = (b_n s_n g_i \dots g_m; g_1 \dots g_m)$ is a circuit that covers $h_n^k G_{n-1}$. As $C_{n-1}(k)$ satisfies the h-property and the $h\tau$-property by Proposition 7 $C_{n-1}(k+1)$ satisfies both properties. We can join $C_{n-1}(k)$ with $C_{n-1}(k+1)$ with the arcs $a_n \to a_n s_n$ and $b_n s_n \to b_n$, and reduce it to a circuit by erasing the arc between a_n and b_n and the arc between

$b_n s_n$ and $a_n s_n$. As the arcs used to join $C_{n-1}(k+1)$ with $C_{n-1}(k)$ are given by vertices belonging to $H\tau$ in $C_{n-1}(k+1)$ and the arcs used to join $C_{n-1}(k+1)$ with $C_{n-1}(k+2)$ are given by vertices belonging to H in $C_{n-1}(k+1)$, we can join the three circuits to form a circuit. We continue with this process until we find a circuit covering each part of G_n, and join the circuits with the arcs given in the recursion. As the arc giving the h-property in the last circuit is not erased it gives the h-property in the final circuit. The arcs giving the $h\tau$-property and the τ-property in the first circuit are not erased, and they give the $h\tau$-property and the τ-property in the final circuit. As we end the circuit by returning to the first circuit and ending it, the last generator is τ. Thus the circuit obtained is an A-circuit, and as we keep the starting vertex from $C_{n-1}(1)$, its starting vertex belongs to H.

$G_3 = \langle \tau, r^6, r^{10}, r^{15}\tau \rangle$



Using the recursive step we will find an A-circuit that covers $D_H = \langle S \rangle$, and this will be the desired Hamilton circuit. □

Remark 17. *As we just said, the Hamilton circuit provided in the theorem is actually an A-circuit.*

In [6] S. Curran showed that if D_n is a dihedral group and $S = \{\tau, \rho_1\tau, \ldots, \rho_m\tau\}$ is a generating set which only contains reflections then if $\overrightarrow{Cay}(\mathbb{Z}_n, S')$ has a Hamilton circuit when $S' = \{\rho_1, \ldots, \rho_m\}$ then $\overrightarrow{Cay}(D_n, S)$ has a Hamilton circuit. We will show that his proof also works in generalized dihedral groups.

Theorem 18. *Let D_H be a generalized dihedral group and $S = \{\tau, h_1\tau, \ldots, h_n\tau\}$ be a set of generators which only contains reflections; then if $\overrightarrow{Cay}(H, S')$ has a Hamilton circuit when $S' = \{h_1, \ldots, h_n\}$ then $\overrightarrow{Cay}(D_H, S)$ has a Hamilton circuit.*

Proof. We will only show that Curran's proof also works in this case.

Suppose that $g_1 \dots g_m$ is a list of generators that produces a Hamilton circuit in $\overrightarrow{Cay}(H, S')$ and let $s_i = g_i\tau \in S$ then the circuit $C = (\tau; \tau s_1 \tau s_2 \dots \tau s_j)$ is a Hamilton circuit in $\overrightarrow{Cay}(D_H, S)$. To prove this we first note that the length of C is exactly $2|H| = |D_H|$, thus the only thing we have to prove is that it does not pass through the same vertex twice. Suppose that it does pass through the same vertex twice; then $\tau\tau s_1 \tau \dots s_j \tau^\alpha = \tau\tau s_1 \tau \dots s_k \tau^\beta$, as the initial vertex is τ and assuming $j < k$. But we have that $s_i\tau = g_i$, and so

$$\begin{aligned}
\tau\tau s_1 \tau \dots s_j \tau^\alpha &= \tau\tau s_1 \tau \dots s_k \tau^\beta \\
g_1 g_2 \dots g_j s_j \tau^\alpha &= g_1 \dots g_{k-1} s_k \tau^\beta \\
g_1 g_2 \dots g_{j-1} g_j \tau^{\alpha+1} &= g_1 \dots g_{k-1} g_k \tau^{\beta+1}.
\end{aligned}$$

But the equality can only occur if $\tau^{\alpha+1} = \tau^{\beta+1}$, and so we have

$$\begin{aligned}
g_1 g_2 \dots g_{j-1} g_j \tau^{\alpha+1} &= g_1 \dots g_{k-1} g_k \tau^{\beta+1} \\
g_1 g_2 \dots g_{j-1} g_j &= g_1 \dots g_{k-1} g_k \\
e &= g_{j+1} \dots g_k,
\end{aligned}$$

but this cannot be, because $g_1 \dots g_m$ gave a circuit. This means that $C = (\tau; \tau s_1 \tau s_2 \dots \tau s_m)$ is a Hamilton circuit in $\overrightarrow{Cay}(D_H, S)$. □

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