LARGE IMAGES OF REDUCIBLE GALOIS REPRESENTATIONS

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ABSTRACT. Given a reducible Galois representation $\overline{\rho} : G_{\mathbb{Q}} \to GL_2(\mathbb{F}_q)$ we show that there exists an irreducible deformation $\rho : G_{\mathbb{Q}} \to GL_2(\mathbb{W}[[T_1, T_2, \ldots, T_r, \ldots]])$ of $\overline{\rho}$ ramified at infinitely many primes, where $\mathbb{W}$ denotes the ring of Witt vectors of $\mathbb{F}_q$. This is a modification of Ramakrishna's result for the irreducible case.

1. INTRODUCTION

In [5] it was shown that one could lift a $\bmod\, p$ representation $\overline{\rho}$ to a power series ring in infinitely many variables which was generalized for totally real fields by [4]. In this paper, we extend these results for a reducible representation $\overline{\rho} : G_{\mathbb{Q}} \to GL_2(\mathbb{F}_q)$, where $\mathbb{F}_q$ is a finite field of residue characteristic p and cardinality $q = p^t$. We use cohomology classes which work for all lifts ρ_n, unlike [2] where their cohomology classes cannot be used to lift from $\bmod\, p$ to $\bmod\, p^2$. This allows us to get an irreducible deformation of a reducible representation in infinitely many variables. The case of a reducible deformation of a residually reducible representation was addressed in [6]. The author hopes to use these methods to generalize other lifting results of Ramakrishna for arbitrary number fields in an ongoing project.

Our main theorem is the following:

Theorem 1.1. *Let* $\overline{\rho} : G_{\mathbb{Q}} \to GL_2(\mathbb{F}_q)$ *where* $\overline{\rho} = \begin{pmatrix} \phi & * \\ 0 & 1 \end{pmatrix}$, *and let* S *be the set of primes containing* p *and* ∞ *and all those at which* $\overline{\rho}$ *is ramified. Suppose:*

- $p \geq 3$,
- $\overline{\rho}$ *is indecomposable,*
- *the* $\mathbb{F}_p$ *span of the elements in the image of* ϕ *is all of* $\mathbb{F}_q$,
- $\phi^2 \neq 1$,
- $\phi \neq \chi, \chi^{-1}$, *where* χ *is the* $\bmod\, p$ *reduction of the cyclotomic character,*

- *for $\overline{\rho}$ odd that $\overline{\rho}|_{G_p}$ is not unramified of the form $\begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix}$, and for $\overline{\rho}$ even that $\overline{\rho}|_{G_p}$ is not $\begin{pmatrix} \chi & 0 \\ 0 & 1 \end{pmatrix}$ or $\begin{pmatrix} \chi^{-1} & * \\ 0 & 1 \end{pmatrix}$, where the $*$ may be trivial.*

Then there exists an irreducible deformation $\rho : G_{\mathbb{Q}} \to GL_2(\mathbb{W}[[T_1, T_2, \ldots, T_r, \ldots]])$ of $\overline{\rho}$ ramified at infinitely many primes, where $\mathbb{W}$ denotes the ring of Witt vectors of $\mathbb{F}_q$.

We start with $\overline{\rho} : G_{\mathbb{Q}} \to GL_2(\mathbb{F}_q)$ and by adding primes to the ramification we lift it successively to $\rho_n : G_{\mathbb{Q}, S_n} \to GL_2(\mathbb{W}[[T_1, \ldots, T_n]]/(p, T_1, \ldots, T_n)^n)$, and define $\rho = \varprojlim_n \rho_n$. If R_n is the deformation ring of ρ_n with m_{R_n} its maximal ideal, then we see that $R_n/m_{R_n}^n = \mathbb{W}[[T_1, \ldots, T_n]]/(p, T_1, \ldots, T_n)^n$. We will add more primes of ramification to S_n and get a new set of primes S_{n+1}, such that the deformation ring associated to S_{n+1} has $R_{n+1}/m_{R_{n+1}}^{n+1}$ as a quotient. This gives us a surjection from $R_{n+1}/m_{R_{n+1}}^{n+1} \twoheadrightarrow R_n/m_{R_n}^n$, which allows us to get the inverse limit $R = \varprojlim_n R_n/m_{R_n}^n$.

2. Notation

We refer the reader to the notation used in [2] but briefly outline some definitions and notations here.

- G_Z is the Galois group over $\mathbb{Q}$ of its maximal extension unramified outside a finite set of primes Z.
- For $w \in Z$, $G_w = \mathrm{Gal}(\overline{\mathbb{Q}}_w/\mathbb{Q}_w)$, where $\mathbb{Q}_w$ is the completion of $\mathbb{Q}$ at w.
- For a $G_{\mathbb{Q}} = \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ module M, $\mathbb{Q}(M)$ is the field fixed by the subgroup of $G_{\mathbb{Q}}$ that acts trivially on M.
- The $\mathbb{G}_m$-dual of M is denoted by M^*.
- For $f \in H^1(G_{\mathbb{Q}}, M)$, we denote by L_f the field fixed by the kernel of the homomorphism $f|_{\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}(M))}$.
- $X = Ad^0(\overline{\rho})$ is the set of trace zero 2×2 matrices over $\mathbb{F}_q$ with Galois action through $\overline{\rho}$ by conjugation.
- Let $K = \mathbb{Q}(X^*)$ which is equivalent to $\mathbb{Q}(X, \mu_p)$.
- For w unramified in a Galois extension $L/\mathbb{Q}$ we denote Frobenius at w by σ_w.
- S is the set of primes containing p, ∞ and all those at which $\overline{\rho}$ is ramified.
- For a character $\kappa : G_{\mathbb{Q}} \to \mathbb{F}_q^*$, we denote by $\mathbb{F}_q(\kappa)$ the module $\mathbb{F}_q$ with Galois action via κ.

3. Trivial primes and the modification of N_v

We modify the lemmas in [5] for a residually reducible representation $\overline{\rho} : G_{\mathbb{Q}} \to GL_2(\mathbb{F}_q)$ using the language of [2] and some ideas from [1]. The following lemmas are used in the next section to find sets of primes that we add to the ramification to remove global obstructions, and cohomology classes associated to these new primes which we use to overcome local obstructions to lifting at each level n. The lemmas

are adaptions of lemmas of Ramakrishna, so we show the modification and outline the rest of the argument.

Definition. Let $\overline{\rho}$ be as in the hypothesis of Theorem 1.1. For v unramified in $\overline{\rho}$ we say v is a trivial prime if:

- v is unramified in $\mathbb{Q}(\overline{\rho})$ and $\overline{\rho}(\sigma_v)$ is trivial, and
- $v \equiv 1 \mod p$.

Since $\overline{\rho}$ is reducible, the Galois module $X = Ad^0(\overline{\rho})$ has a filtration of Galois stable $\mathbb{F}_q$-subspaces of the form $U_1 = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$, $U_2 = \begin{pmatrix} a & b \\ 0 & -a \end{pmatrix}$, $U_3 = \begin{pmatrix} a & b \\ c & -a \end{pmatrix}$, while the Galois module X^* has a filtration of the $\mathbb{F}_q$-subspaces $V_1 = (X/U_2)^*$, $V_2 = (X/U_1)^*$, $V_3 = X^*$. For a subquotient M of X or X^*, the ϕ, trivial, ϕ^{-1}, $\chi\phi$, ϕ, $\chi\phi^{-1}$ eigenspaces are the eigenspaces under the prime to p action of $\mathrm{Gal}(\mathbb{Q}(\phi, \mu_p)/\mathbb{Q})$ under a splitting of the long exact sequence

$$1 \to \mathrm{Gal}(K/\mathbb{Q}(\phi, \mu_p)) \to \mathrm{Gal}(K/\mathbb{Q}) \to \mathrm{Gal}(\mathbb{Q}(\phi, \mu_p)/\mathbb{Q}) \to 1.$$

Definition. For any $M \in \{U_1, U_2, U_3, V_1, V_2, V_3\}$ and Z a finite set of primes containing S we define $\text{Ш}_Z^i(M)$ to be the kernel of the map

$$H^i(G_Z, M) \to \oplus_{v \in Z} H^i(G_v, M).$$

Definition. Let N_w be a subgroup of $H^1(G_w, M)$ and let N_w^* be its annihilator in $H^1(G_w, M^*)$ under local Tate duality. Let $N = \{N_w\}_{w \in \mathbb{Z}_p}$. The Selmer group $H_N^1(G_Z, M)$ is the kernel of the restriction map

$$H^1(G_Z, M) \to \oplus_{w \in Z} H^1(G_w, M)/N_w.$$

Let $N^* = \{N_w^*\}_{w \in \mathbb{Z}_p}$. We define the dual Selmer group $H_{N^*}^1(G_Z, M^*)$ as the kernel of the other restriction map:

$$H^1(G_Z, M^*) \to \oplus_{w \in Z} H^1(G_w, M^*)/N_w^*.$$

Definition. We say an element $f \in H^1(G_Z, X)$ (resp. $\psi \in H^1(G_Z, X^*)$) has rank d if d is the smallest number such that f (resp. ψ) is in the image of the map $H^1(G_Z, U_d) \to H^1(G_Z, X)$ (resp. $H^1(G_Z, V_d) \to H^1(G_Z, X^*)$).

Lemma 3.1. *Let M be any of the subspaces $\{U_1, U_2, U_3, V_1, V_2, V_3\}$, then there exists a finite set Q_1 of trivial primes such that $\text{Ш}_{S \cup Q_1}^1(M) = 0$.*

Proof. We mimic the proof of [2, Proposition 13] and outline the argument.

Let $\psi \in H^1(G_{\mathbb{Q}}, X)$ and L_ψ be the field fixed by the kernel of ψ. Let P be the subgroup of $G_{\mathbb{Q}}$ that acts trivially on M and $H = \mathrm{Gal}(L_\psi/\mathbb{Q})$. By [2, Proposition 8], we can assume that $H^1(\mathrm{Gal}(\mathbb{Q}(M)/\mathbb{Q}), M)$ is trivial. We have the inflation-restriction sequence

$$0 \to H^1(H/P, X^P) \to H^1(H, M) \to H^1(P, M)^{H/P}.$$

Now, $H/P = \mathrm{Gal}(\mathbb{Q}(M)/\mathbb{Q})$ and P acts trivially on X, so $H^1(H/P, M^P) = H^1(\mathrm{Gal}(\mathbb{Q}(M)/\mathbb{Q}), M)$, which was assumed to be trivial. So a non-trivial $\psi \in$

$H^1(H, M)$ gives rise to a non-trivial element of $H^1(P, M)^{H/P}$ which is $\mathrm{Hom}(P, M)^{H/P}$ as P acts trivially on M. This shows that L_ψ is a non-trivial extension of $\mathbb{Q}(M)$. If $\psi \in \mathrm{III}^1_S(M)$, then we choose a trivial prime q such that it splits completely from $\mathbb{Q}$ to $\mathbb{Q}(M)$ but not from $\mathbb{Q}(M)$ to L_ψ, which means that $\psi|_{G_q} \neq 0$, so $\psi \notin \mathrm{III}^1_{S \cup \{q\}}(M)$. As $H^1(G_S, M)$ is finite, we repeat this procedure and get a finite set of trivial primes Q_1 such that $\mathrm{III}^1_{S \cup Q_1}(M) = 0$. $\square$

Let $(z_v)_{v \in S \cup Q_1} \notin \oplus_{v \in S \cup Q_1} N_v \oplus \psi_{S \cup Q_1}(H^1(G_{S \cup Q_1}, X))$ be a set of cohomology classes which we will use eventually to overcome obstructions to lifting in the next section.

Lemma 3.2. *Let Q_1 be as in Lemma 3.1. There exists a Cebotarev class L of trivial primes such that*

- *$\beta|_{G_v} = 0$ for all $\beta \in H^1(G_{S \cup Q_1}, U_i^*)$ for $i = 1, 2$ and for all $\beta \in H^1(G_{S \cup Q_1}, X)$.*
- *There exists an $\mathbb{F}_p$-basis $\{\psi, \psi_1, \ldots, \psi_r\}$ of $H^1(G_{S \cup Q_1}, X^*)$ such that $\{\psi_1, \ldots, \psi_r\}$ is a basis of $\psi_{S \cup Q_1}^{*-1}(\mathrm{Ann}(z_w)_{w \in S \cup Q_1})$, $\psi|_{G_v} \neq 0$ and $\psi_i|_{G_v} = 0$ for all $i \geq 1$.*

Furthermore, there is for each $v \in L$, a rank 3 element $h^v \in H^1(G_{S \cup Q_1 \cup \{v\}}, X)$ and a decomposition group above v such that $h^v|_{G_w} = (z_w)_{w \in S \cup Q_1}$ and $h^v(\tau_v) = \begin{pmatrix} 0 & 0 \\ s & 0 \end{pmatrix}$ with $s \neq 0$.

Proof. The difference between the above lemma and [2, Proposition 34] is that we have added the additional condition of $\beta|_{G_v} = 0$ for all $\beta \in H^1(G_{S \cup Q_1}, X)$, where X corresponds to U_3 in the notation above. This means that we need the prime v to split completely in the ϕ, ϕ^{-1} and identity eigenspaces which are disjoint from the χ/ϕ eigenspace of U_1^* and the χ/ϕ and χ eigenspaces of U_2^*. The modified definition of trivial primes imposes only splitting conditions and the only non-splitting condition in the hypothesis above is in the $\chi\phi$ eigenspace of $\mathrm{Gal}(K_\psi/K)$, none of which are in U_1^*, U_2^* and X. Now, following the argument of [2, Proposition 34] we see that v comes from a Cebotarev condition. $\square$

As $h^v(\tau_v) = \begin{pmatrix} 0 & 0 \\ s & 0 \end{pmatrix}$ with $s \neq 0$, we define the sets C_v and N_v of [1] to be the conjugates by the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ for the primes v that we add.

We cannot control the behavior of h^v at σ_v, so we add a pair of primes v_1, v_2 such that $h = -h^{v_1} + 2h^{v_2}$ has the appropriate image of Frobenius and $h|_{G_w} = z_w$ for $w \in S \cup Q_1$. Altering the definition of trivial primes still allows us to use the same techniques of [2] so we can use the following result ([2, Theorem 41]).

Theorem 3.3. *There is a set of two primes $\{v_1, v_2\}$ coming from the Cebotarev class L in the previous lemma such that for $h = -h^{v_1} + 2h^{v_2}$ we can choose the values of $h(\sigma_{v_i})$ arbitrarily for $i = 1, 2$.*

4. MAIN THEOREM AND ITS PROOF

Let C_l be the set of deformation classes of $\bar{\rho}$ to $\mathbb{W}$ satisfying $\rho(\sigma_l) = \begin{pmatrix} l & 0 \\ 0 & 1 \end{pmatrix}$
and $\rho(\tau_l) = \begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix}$.

We define $u_1, u_2 \in H^1(G_l, X)$ by:

$$u_1(\sigma_l) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad u_1(\tau_l) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$u_2(\sigma_l) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad u_2(\tau_l) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

Note that these two cohomology classes are the same as in [2]. We refer the reader to the calculations of [1, Lemma 4.1] to produce the third cohomology class u_3 to get a three dimensional subspace N_l which preserves C_l.

Recall that R_n is the deformation ring of ρ_n with m_{R_n} its maximal ideal. We assume that there exists $\rho_n : G_{S_n} \to GL_2(R_n/m_{R_n}^n)$, $\text{III}_{S_n}^2(X) = 0$ and $\dim H_N^1(G_{S_n}, X) = n$. By Theorem 3.3 we can find a set of primes B such that $\dim H_N^1(G_{S_n \cup B}, X) = n + 1$ (we simply choose the $\alpha_i \in C_{v_i}$ in the proof of Theorem 3.3). Let U be the deformation ring and ρ_U be the deformation associated to the augmented set $S_n \cup B$, with the deformation conditions (N_v, C_v). If B consists of primes such that $\rho_n|_{G_v} \in C_v$ for $v \in B$, we have a surjection $\phi : U \twoheadrightarrow R_n/m_{R_n}^n$ and we follow the argument as in [5] or [4].

If $\rho_n|_{G_v} \notin C_v$ for $v \in B$, then we choose a set of cohomology classes $(z_v)_{v \in S_n \cup B}$ such that the action of z_v on $\rho_n|_{G_v}$ overcomes the local obstructions at $v \in S_n \cup B$. By Theorem 3.3 we can find a set A of two primes and a cohomology class h such that:

- $\tilde{\rho}_n = (I + p^n h)\rho_n|_{G_q} \in C_q$ for $q \in A$ (no new obstructions at A),
- $h|_{G_v} = z_v$, for $v \in S_n \cup B$ (h overcomes local obstructions at $S_n \cup B$).

We now show that adding this set of primes A does not alter the dimension of the Selmer groups, hence does not add more variables to the ring of power series.

Lemma 4.1. *For a set $A = \{v_1, v_2\}$ of two primes chosen as in Theorem 3.3 and $(z_v)_{v \in S_n \cup B} \notin \oplus_{v \in S_n \cup B} N_v \oplus \psi_{S_n \cup B}(H^1(G_{S_n \cup B}, X))$ we have $H_N^1(G_{S_n \cup B}, X) = H_N^1(G_{S_n \cup B \cup A}, X)$.*

Proof. We adapt the argument of [5, Proposition 4.1].

Recall that in Lemma 3.2 the trivial primes v were chosen so that $\beta \in H^1(G_T, X) \Rightarrow \beta|_{G_v} = 0$. As N_v is a three dimensional subspace including the zero cocyle u_2, we see that $\beta|_{G_v} = 0 \Rightarrow \beta \in N_v$. Thus, $H_N^1(G_{S_n \cup B}, X) \subset H_N^1(G_{S_n \cup B \cup A}, X)$.

Any element of $H_N^1(G_{S_n \cup B \cup A}, X) \setminus H_N^1(G_{S_n \cup B}, X)$ necessarily looks like $f + \alpha_1 h^{v_1} + \alpha_2 h^{v_2}$, where $f \in H^1(G_{S_n \cup B}, X)$ and h^{v_i} are as in Theorem 3.3. Since $\alpha_1 h^{v_1} + \alpha_2 h^{v_2}|_{G_v} = (\alpha_1 + \alpha_2)z_v \in f + N_v$ for $v \in S_n \cup B \cup A$, we see that $\alpha_1 + \alpha_2 = 0$. We know that $\alpha_1(h^{v_1} - h^{v_2})|_{G_q} = 0$ for all $q \in S_n \cup B$, which means that $f|_{G_q} \in N_q$ for all $q \in S_n \cup B \Rightarrow f \in H_N^1(G_{S_n \cup B}, X)$. We also know that $f|_{G_{v_i}} = 0$ for

$i = 1, 2$, so $f \in H^1_N(G_{S_n \cup B \cup A}, X)$. Thus, $\alpha_1(h^{v_1} - h^{v_2}) \in H^1_N(G_{S_n \cup B \cup A}, X) \Rightarrow$ $\alpha_1(h^{v_1} - h^{v_2})|_{G_{v_i}} \in N_{v_i}$, for $i = 1, 2$. We now look at the construction of the h^{v_i} in the proof of the previous lemma to get a contradiction.

If $\rho_n|_{G_{v_i}} \notin C_{v_i}$, we choose $h|_{G_{v_i}} = -h^{v_1} + 2h^{v_2} \notin N_{v_i}$ for $i = 1, 2$. This implies that $-A + 2E \notin N_{v_1}$, while we have that $h^{v_1} - h^{v_2}|_{G_{v_i}} \in N_{v_1} \Rightarrow A - E \in N_{v_1}$. Combining these two conditions we get that $A, E \notin N_{v_1}$ so $\alpha_1 = 0$ and $f \in H^1_N(G_{S_n \cup B \cup A}, X)$, which is a contradiction. A similar argument works for N_{v_2}.

If $\rho_n|_{G_{v_i}} \in C_{v_i}$ and $h^{v_i}(\sigma_{v_i}) \notin N_{v_i}$ i.e., $A \notin N_{v_1}$, then using the fact that $A - E \in N_{v_1}$ and $-A + 2E \in N_{v_1}$, we get that $A \in N_{v_1}$, which is a contradiction.

(Note that if A consists of only one prime, then the proof is exactly the same as in the first part of [5, Proposition 4.1].) □

Let $\tilde{W}$ be the deformation ring and $\rho_{\tilde{W}}$ associated to the augmented problem with deformation conditions (N_v, C_v). As $\tilde{\rho}_n|_{G_q} \in C_q$ for $q \in A$ we have a surjection $\phi : \tilde{W} \twoheadrightarrow R_n/m^n_{R_n}$, which means that for some I_1, we have $\tilde{W}/I_1 = R_n/m^n_{R_n}$. As $\dim H^1_N(G_{S_n \cup B}, X) = \dim H^1_N(G_{S_n \cup B \cup A}, X) = n + 1$, we see that as a ring $\tilde{W}$ consists of power series of $(n + 1)$ variables. Thus, for some I_2, $\tilde{W}/I_2 = \mathbb{F}_q[[T_1, \ldots, T_{n+1}]]/(T_1, \ldots, T_{n+1})^2$. Let $I = I_1 \cap I_2$, and define $W_0 = \tilde{W}/I$.

Our goal is to get a deformation ring which has $R_{n+1}/m^{n+1}_{R_{n+1}}$ as a quotient. If W_0 is such a deformation ring, we are done. If not, we get a sequence

$$R_{n+1}/m^{n+1}_{R_{n+1}} \twoheadrightarrow \cdots \twoheadrightarrow W_1 \twoheadrightarrow W_0$$

where the kernel at each stage has order p. We add more primes of ramification to $S_n \cup B \cup A$ so that the augmented deformation ring has W_1 as a quotient and keep iterating to get our required deformation ring.

As W_0 is a quotient of $\tilde{W}$, we let ρ_{W_0} be the deformation induced by $\rho_{\tilde{W}}$. As $\rho_{W_0}|_{G_v} \in C_v$ for $v \in S_n \cup B \cup A$ we can lift ρ_{W_0} to W_1. Let us call this deformation ρ_{W_1}. Iterating the same argument as for ρ_n we can lift ρ_{W_1} to W_2 by adding a suitable set of primes A_1 to the set of ramification allowing us to eventually find a deformation that has $R_{n+1}/m^{n+1}_{R_{n+1}} = \mathbb{W}[[T_1, \ldots, T_{n+1}]]/(p, T_1, \ldots, T_{n+1})^{n+1}$ as a quotient. Now we are in a position to state the final theorem.

Theorem 4.2. *There exists an irreducible deformation of $\bar{\rho}$, ramified at infinitely many primes, $\rho : G_{\mathbb{Q}} \to GL_2(\mathbb{W}[[T_1, T_2, \ldots, T_r, \ldots]])$.*

Proof. We let $\rho = \varprojlim_n \rho_n$ and see that at each stage n,

$$\operatorname{Im} \rho_n \supseteq GL_2(\mathbb{W}[[T_1, \ldots, T_n]]/(p, T_1, \ldots, T_n)^n).$$

Hence, we get our desired deformation. By [2, Corollary 43], the deformation is irreducible. □

5. Concluding remarks

- In [4], one could not generalize the results of [5] for all number fields. One of the problems in using our definition of trivial primes is that when one adds them to the ramification set to solve the local condition property

(finding an h^v such that $h^v|_{G_w} = (z_w)_{w \in Z}$), the behavior at inertia is hard to control. In the reducible case one can use the subspaces U_i to find a suitable h^v, but in the irreducible case it is hard to guarantee the behavior of h^v at inertia.

- In [5], the image of the deformation is full, i.e., ρ contains $SL_2(\mathbb{Z}_p[[T_1, T_2, \ldots, T_r, \ldots]])$ but requires that the image of the residual representation $\overline{\rho}$ contains $SL_2(\mathbb{Z}/p\mathbb{Z})$, which is not true in our case. Hence, we do not get that the image of our deformation is full.

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